

Christiane H. Schmickler · Hans-Werner Hammer · Emiko
Hiyama

Trimer and Tetramer Bound States in Heteronuclear Systems

Received: date / Accepted: date

Abstract The Efimov effect in heteronuclear cold atomic systems is experimentally more easily accessible than the Efimov effect for identical atoms, because of the potentially smaller scaling factor. We focus on the case of two or three heavy identical bosons and another atom. The former case was recently observed in a mixture of ^{133}Cs and ^6Li atoms.

We employ the Gaussian Expansion Method as developed by Hiyama, Kino et al. [7]. This is a variational method that uses Gaussians that are distributed geometrically over a chosen range. Supplemental calculations are performed using the Skorniakov-Ter-Martirosian equation.

Blume et al. [1] previously investigated the scaling properties of heteronuclear systems in the unitary limit and at the three-body breakup threshold. We have completed this picture by calculating the behaviour on the positive scattering length side of the Efimov plot, focussing on the dimer threshold.

Keywords Efimov physics · Four-boson system · Universality · Heteronuclear systems

1 Introduction

The Efimov effect has been intensively investigated in recent years, both theoretically and experimentally. It was predicted by Efimov [4] that in a system with a scattering length a that is large compared to the typical interaction length of the system, a series of universal three-body bound states can be found with a fixed factor between them. In the unitary limit, $\frac{1}{a} = 0$, the energies of consecutive states fulfil

$$\frac{E_{n+1}}{E_n} = \lambda, \quad (1)$$

where λ is a universal factor that does not depend on details of the interaction. It does depend however on the number of interacting pairs and the mass imbalance, among other things. The typical spectrum resulting from this is usually depicted in a so-called Efimov plot as presented in Fig. 1.

The most easily accessible systems experimentally are systems of two species of atoms (heavy bosons H and one light atom L) with a large mass imbalance. There the Efimov effect occurs as well and the large mass imbalance leads to a comparatively small factor λ between consecutive states [2].

C.H. Schmickler
Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
RIKEN Nishina Center, RIKEN, Saitama 351-0198, Japan
E-mail: schmickler@theorie.ikp.physik.tu-darmstadt.de

H.-W. Hammer
Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

E. Hiyama
RIKEN Nishina Center, RIKEN, Saitama 351-0198, Japan

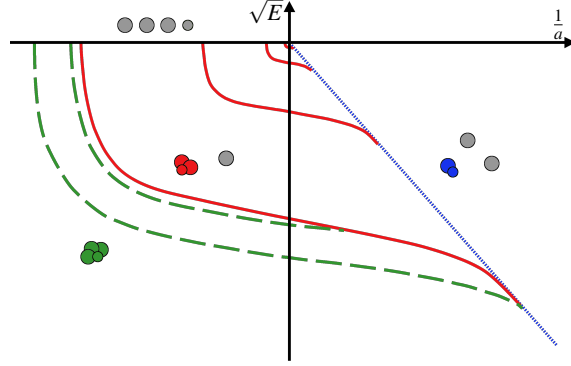


Fig. 1 Schematic Efimov plot of the mass imbalanced four-body system. The H_2L trimers are the solid red lines, the H_3L tetramers are the dashed green lines and the HL dimer threshold is the dotted blue line. Since we assume no interaction between the H atoms, there is only one dimer.

In addition to the standard three-body effect, there has also been a lot of interest in expanding the universal picture into the N -body sector. In systems of identical bosons, for example, two universal tetramers attached to the lowest Efimov trimer have been found [5][8][3].

Bringing these two aspects together, Blume and Yan have studied four-body states in a heteronuclear system [1]. They focussed on the negative scattering length side of the Efimov spectrum, because this is the side most experiments use for their observations.

To complete this picture we have studied the side of positive scattering length. This side can also be seen experimentally, although experiments are complicated by the existence of the shallow dimer. Experiments typically measure atomic losses due to resonant recombination at the threshold, so we have focussed on the region near the dimer threshold using full four-body calculations with the Gaussian Expansion Method [7].

2 Method

The Gaussian Expansion Method employs the Rayleigh-Ritz Variational Principle with base functions that are selected via geometric progression between a minimum and a maximum range. It uses various representations of the system in Jacobi Coordinates, which we call rearrangement channels, to cover different possible angular momentum couplings and arrangements. The total wavefunction Ψ the Schrödinger Equation $(H - E)\Psi = 0$ is projected onto is composed of a set of Gaussian functions for each Jacobi Coordinate, so in the three-body case we have

$$\Psi_{JM}^{\text{trimer}} = \sum_{c=1}^2 \sum_{n_c, N_c} \sum_{\ell_c, L_c} C_{n_c \ell_c N_c L_c}^{(c)} [\phi_{n_c \ell_c}^{(c)}(\mathbf{r}_c) \psi_{N_c L_c}^{(c)}(\mathbf{R}_c)]_{JM}, \quad (2)$$

with

$$\phi_{nlm}(\mathbf{r}) = Y_{lm}(\hat{\mathbf{r}}) N_{nl} r^l e^{-(r/r_n)^2} \quad \text{and} \quad \psi_{NLM}(\mathbf{R}) = Y_{LM}(\hat{\mathbf{R}}) N_{NL} R^L e^{-(R/R_N)^2}. \quad (3)$$

The relevant rearrangement channels for the trimer are shown in Fig. 2.

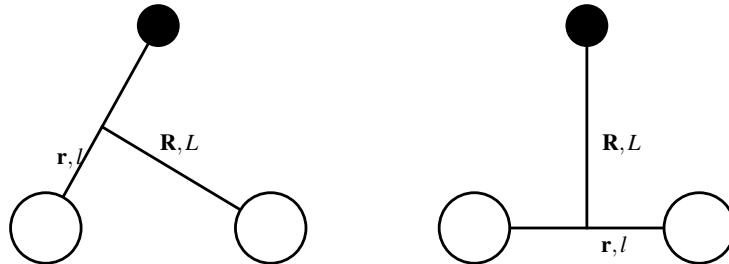


Fig. 2 The two possible rearrangement channels for the H_2L three-body system. The big white circles represent the heavy bosons H , while the small black circle represents the light atom L .

The r_n are given by

$$r_n = r_{\min} \left(\frac{r_{\max}}{r_{\min}} \right)^{\frac{n-1}{n_{\max}-1}}, \quad n = 1, \dots, n_{\max}, \quad (4)$$

and analogously for R_N , which means for each Jacobi coordinate we have 3 parameters, r_{\max} , r_{\min} and n_{\max} . More details about this method can be found in [7]. Using several rearrangement channels and relative angular momentum configurations, the parameter space increases rapidly for higher N systems. Because of this, systematic sampling of base functions was impractical for the four-body system and we employed random sampling within relatively broad ranges to find optimized base functions.

3 Interaction

Because there is no reason why the HL interaction should be resonant at the same point as the HH interaction, we can safely assume $a_{HH} \ll a_{HL}$ and neglect a_{HH} . Adding an interaction between the heavy bosons only moves the whole Efimov spectrum up or down [9], which we can do independently by tuning our three-body force.

As we want to focus on universal behaviour we choose effective potentials, which have the added benefit of being numerically well behaved. To keep the calculated energies in the range of validity of the effective theory we add a repulsive three-body force. The inverse scattering length $\frac{1}{a}$ is roughly linear in the two-body potential strength, which is how we tune it.

$$V_{iN} = V_0 \exp\left(-\frac{r_{iN}^2}{2r_0^2}\right), \quad W_{ijN} = W_0 \exp\left(-\frac{r_{ij}^2 + r_{jN}^2 + r_{iN}^2}{16r_0^2}\right). \quad (5)$$

Here i and j run over the heavy (H) bosons of mass M from $1, \dots, N-1$, while the N th atom (L) is light and of mass m . The r_{ij} are the distances between atoms i and j . The natural energy scale of the problem is

$$E_s = \frac{1}{2r_0^2} \frac{m+M}{Mm}, \quad (6)$$

where r_0 is the effective range of the two-body potential. We have ensured $E \ll E_s$ throughout our calculations.

4 Results

Our main result is that trimer and tetramer vanish at almost the same point for all systems we have looked at, which were ${}^7\text{Li}$ - ${}^6\text{Li}$, ${}^{87}\text{Rb}$ - ${}^7\text{Li}$ and ${}^{133}\text{Cs}$ - ${}^6\text{Li}$ mixtures.

We have taken a closer look at the behaviour of trimer and tetramer near the dimer threshold. Our results for the two most extreme cases, almost no mass imbalance (${}^7\text{Li}$ - ${}^6\text{Li}$) and very high mass imbalance (${}^{133}\text{Cs}$ - ${}^6\text{Li}$), are shown in Fig. 3. It can be seen that in the (7/6) case the states approach the threshold very slowly, whereas for the (133/6) case there is a slightly steeper slope. Also, the difference c_d between the point where the trimer vanishes, c_3 , and the

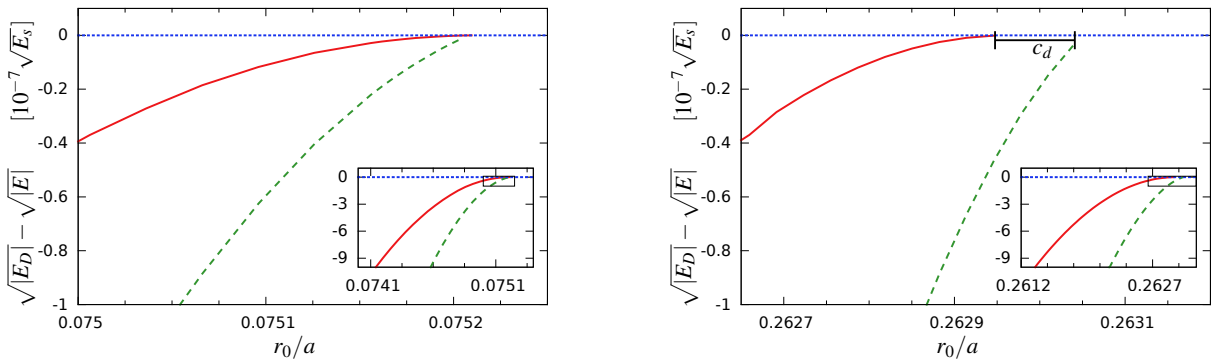


Fig. 3 Region of the Efimov plot very close to the dimer threshold. We display the difference to the dimer threshold (blue dotted line) of the H_2L trimer (red solid line) and the H_3L tetramer (green dashed line) for two different systems, ${}^7\text{Li}$ - ${}^6\text{Li}$ mixture in the left panel and ${}^{133}\text{Cs}$ - ${}^6\text{Li}$ mixture in the right one. The insets are the zoomed out versions of the plots. We have included an illustration of the definition of c_d in the right plot.

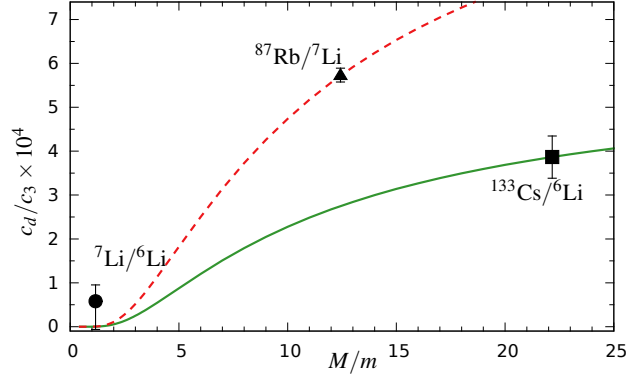


Fig. 4 The difference c_d between trimer and tetramer crossing point in units of the trimer crossing point c_3 as a function of the mass ratio M/m . The solid green (dashed red) lines are effective three-body calculations using the Skorniakov-Ter-Martirosian equation and fitted to reproduce the 133/6 (87/7) point.

point where the tetramer vanishes, c_4 , as shown in the right panel of Fig. 3, is much larger for the (133/6) case than for the (7/6) case. To quantify this, we have extracted these numbers for the three systems we have investigated and plotted them in Fig. 4.

To help interpret this picture, we carried out effective three-body calculations of the dimer-atom-atom system using the Skorniakov-Ter-Martirosian (STM) equation as described in [6]. This picture is valid near the dimer threshold where the trimer and tetramer are very shallow. In this region the scattering length between HL dimer and H atom can be estimated by the inverse trimer binding energy and is very large ($a_{AD}/r_0 \approx 10^7$) so an effective three-body treatment is justified. The STM equation has one free parameter which we fitted to reproduce either the data point belonging to the (133/6)-system, or the (87/7) system. In both cases, the data point of the (7/6)-system is reproduced, but the (133/6) and (87/7) points cannot be fitted simultaneously. This might mean that there is non-universal behaviour governing the c_d/c_3 value. But it could also simply be the case that the effective three-body calculation does not capture enough of the four-body system to yield accurate predictions.

5 Outlook

As a next step, we plan to investigate the dependence of c_d on the potential shape and the strength of the three-body potential to rule out the possibility of non-universal behaviour. Additionally, calculating more data points for Fig. 4 might deepen the understanding of the underlying functional dependence.

Acknowledgements We thank Doerte Blume for useful discussions and comments. Part of the calculations were carried out at YITP of Kyoto University. C.H. Schmickler thanks RIKEN for providing support under the IPA program.

References

1. Blume D, Yan Y (2014) Generalized Efimov Scenario for Heavy-Light Mixtures. Phys Rev Lett 113:213201 arXiv:1410.2314
2. Braaten E, Hammer HW (2006) Universality in Few-body Systems with Large Scattering Length. Phys Rep 428:259–390 arXiv:cond-mat/0410417v3
3. Deltuva A (2013) Properties of universal bosonic tetramers. Few Body Syst 54:569–577 arXiv:1202.0167v1
4. Efimov V (1970) Energy levels arising from resonant two-body forces in a three-body system. Phys Lett B 33:563–564 DOI:10.1016/0370-2693(70)90349-7
5. Hammer HW, Platter L (2007) Universal Properties of the Four-Body System with Large Scattering Length. Eur Phys J A 32:113–120 arXiv:nucl-th/0610105v2
6. Helfrich K, Hammer HW, Petrov DS (2010) Three-body problem in heteronuclear mixtures with resonant interspecies interaction. Phys Rev A 81:042715 DOI:10.1103/PhysRevA.81.042715
7. Hiyama E, Kino Y, Kamimura M (2003) Gaussian expansion method for few-body systems. Prog Part Nucl Phys 51:223–307 DOI:10.1016/S0146-6410(03)90015-9
8. von Stecher J, D’Incao JP, Greene CH (2009) Signatures of universal four-body phenomena and their relation to the Efimov effect. Nat Phys 5:417–421 DOI:10.1038/nphys1253
9. Wang Y, Laing WB, von Stecher J, Esry BD (2012) Efimov Physics in Heteronuclear Four-Body Systems. Phys Rev Lett 108:073201 arXiv:1109.1581